

Device on basis of a bent crystal with variable curvature for particle beams steering in accelerators.

A.G.Afonin, V.T.Baranov, M.K.Bulgakov, Yu.A.Chesnokov, P.N.Chirkov, E.V.Lobanova,
I.S.Lobanov, A.N.Lunkov, V.A.Maishev, I.V.Poluektov, Yu.E.Sandomirskiy, I.A.Yazynin.
Institute for High Energy Physics, 142281, Protvino, Russia

Recently it was proposed to apply a bent single crystal with decreasing curvature instead of uniform bending for improvement of extraction and collimation of a circulating beam in particle accelerators. In the given paper created crystal devices with a variable curvature, realizing this idea are described. Results of measurement of curvature along a crystal plate are informed. It is shown, that with the help of the developed devices it is possible to carry out also high energy beam focusing. The mathematical description of this process is proposed.

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I. INTRODUCTION

At present, the collimation and extraction of a circulating beam using coherent phenomena in oriented single crystals are examined at several large accelerators. Pioneering works [1–4] at the U-70 IHEP accelerator indicate that, in short bent silicon crystals, channeling can increase the efficiency of the extraction and collimation of the beam up to 85%. This possibility has been confirmed at the SPS collider (CERN) [5] and at the Tevatron (Fermilab) [6]. In view of the commissioning of the Large Hadron Collider (LHC) and the problem of an increase in its luminosity, the problem of improving the beam collimation efficiency is of particular importance [7]. In [8] it was proposed to apply the bent single crystal with decreasing curvature instead of uniform bending for improvement of extraction and collimation of a circulating beam in particle accelerators. Presence of decreasing curvature in a crystal leads to suppression of dechanneling and thus way improves parameters of extraction/collimation of a circulated beam. In the given work the created devices with variable curvature are described. Results of measurement of curvature along a crystal plate are informed. It is shown, that with the help of the developed devices it is possible to carry out also high energy beam focusing.

II. THE SCHEME OF CRYSTAL BENDING WITH VARIABLE CURVATURE

For realization of effective extraction of a proton beam from U-70 [1–4] it was necessary to find a challenge decision how to bend the short silicon crystals on small angles. With this purpose a way [1, 4] of bending of a crystal as narrow strips by length of ~ 2 mm along the beam and 40 mm in height has been developed based on use of anisotropic properties of crystal lattices. From the theory of elasticity it is known, that at a bend of a crystal plate in longitudinal direction in an orthogonal direction there are the deformations accepting saddle or barrel form depending on concrete anisotropic properties of a material and orientation of a crystal (see for example [9], page 85). In silicon single crystals the greatest orthogonal deformations are formed at orientation (111) and get the saddle form (see Fig. 1). Thus the bend of a crystal on height on ~ 100 mrad angle provides in an orthogonal direction the bending angle of about 1 mrad sufficient for particle extraction from accelerator [4].

The subsequent precision experiments executed on beam line 8 in northern CERN area with microstrip detectors, have confirmed, that similar crystals effectively deflect 400 GeV protons and are uniformly bent [10]. We have applied the same method of a bend of a silicon strip to a nonuniform bend of a crystal, but used a plate of trapezoidal cross-section (Fig. 1b). In this case due to falling down thickness of a strip different mechanical tensions in crystal material along the horizontal direction (on coordinate z) are created, that leads to decreasing the bending curvature as approaching the narrow side of a trapeze. Empirically picked up tangent of an inclination of a lateral face of a trapeze has allowed to reach the necessary gradient of curvature.

III. MEASUREMENT OF THE BENDING SHAPE OF A CRYSTAL

In Fig. 2a is shown a photo of proposed crystal device. The silicon crystal plate 1 of trapezoidal cross-section is bent in a longitudinal direction by the metal holder 2. With the help of the screw 3 it is provided vertical, on coordinate y (longitudinal) bend of a crystal plate on 20 mrad. Screws 4 eliminate possible twisting a plate on height (eliminate

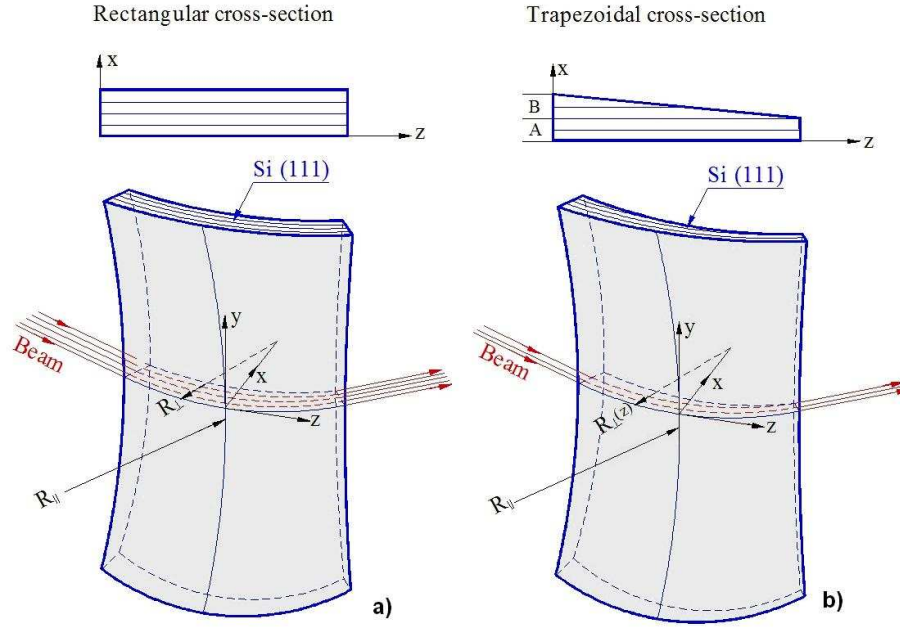


FIG. 1: The scheme of a uniform bend of a crystal - strip of rectangular cross-section. b - a nonuniform bend of a plate of a trapezoidal cross-section.

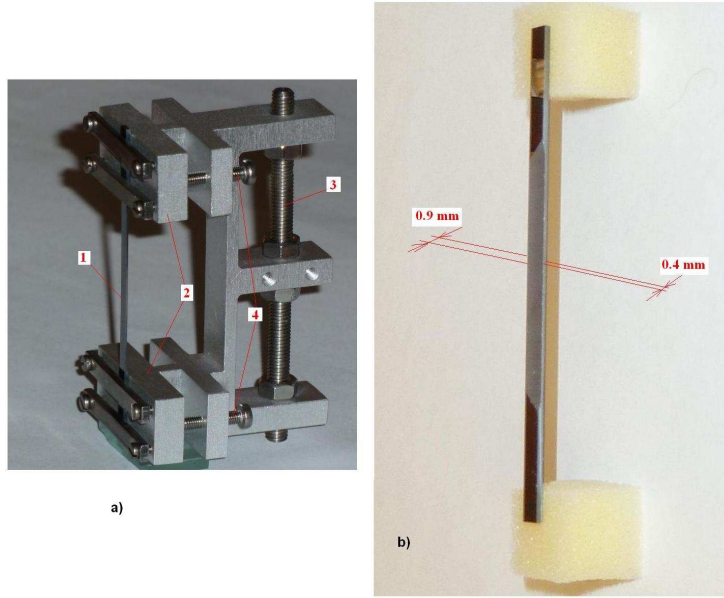


FIG. 2: a - photo of the crystal device, b - view of a crystal plate from the back face.

"effect of a propeller"). Such design of the bending holder for the first time has been offered by Yu.Chesnokov in [1] and then modified in several publications by various researchers.

The image of a crystal from the back side is shown in Fig. 2b, where the processed oblique face of trapezoidal cross-section is visible. The crystal plate which has been cut out initially lengthways crystalline plane (111) in form of a parallelepiped with the sizes $(xyz) = (0.9 \times 70 \times 3)$ mm, in the central part is processed with a slope on a trapeze with a size $x_1 = 0.9$ mm at a forward end face up to $x_2 = 0.4$ mm at a back end face. The shape of a horizontal bend of a crystal (along coordinate z) was measured by the laser device using scheme described in ([9], page 86). Results of measurement of the bending shape of a plate are displayed in Fig. 3.

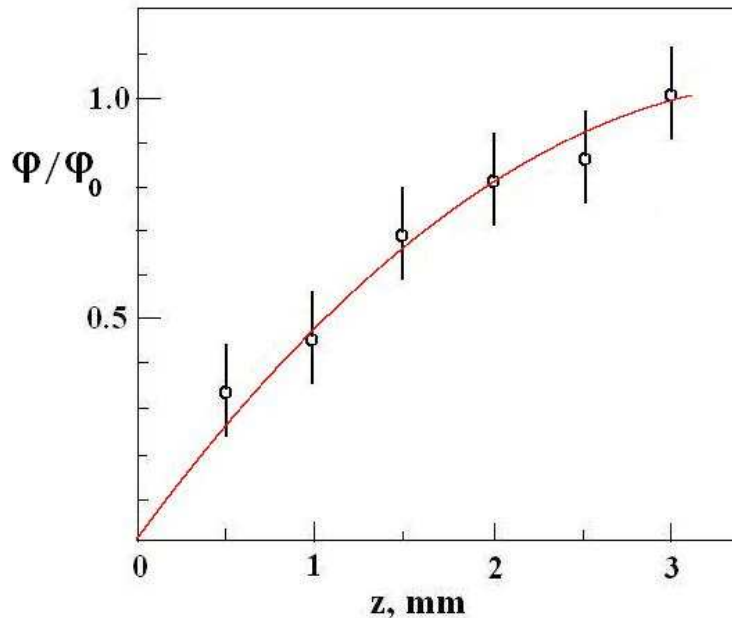


FIG. 3: Results of measurements of the bending angle as a function of z -coordinate (symbols) and its approximation with the help of Eq. (6).

The full angle of a bend is equal $\varphi_0 = 0.3$ mrad. Changing of bending radius along coordinate z can be approximated by the dependence:

$$R(z) = \frac{R_0}{1 - Cz/L_0}, \quad (1)$$

where R_0 is the bending radius at $z = 0$, L_0 is the single crystal thickness and $C = 0.8$ is the constant value. The calculated bending angle $\varphi(z) = \int_0^z dz/R(z)$ (see the curve in Fig. 3) is in a good agreement with the measurements.

The realized parameters of a crystal device approach for an optimum deflection of particles with multi-GeV energy, experiments with which are carried out in IHEP and CERN [11, 12]. It is necessary to note, because of trapezoidal cross-section of a crystal, on a full angle are bent only crystalline planes marked on fig.1b as area A. The planes shown as an area B have the different bending angles according to the length of the oblique side of a trapeze. At crystal extraction of a beam from accelerator only the area A of a crystal is involved in the process, as high energy particles will penetrate into a crystal (on coordinate x) on distance no more than 100 micron (for the case of the LHC energy [13]).

At the same time, the area B of a crystal can be used for focusing of the particle trajectories due to the difference of bending angles depending on coordinate x of incoming particles. For the task of focusing the area B can be expand, changing the cross-section of a crystal up to triangle as shown in fig.5. It is necessary to note, experiments on focusing of high energy particle trajectories by a crystal with the oblique end face were carried out intensively in the beginning of 90-ties [14, 15]. Novelty of application of the suggested crystal device consists in simplicity of a design and absence of superfluous substance around of a working crystal (devices [14] had massive details around of a crystal).

IV. EXAMPLES OF APPLICATIONS OF THE DEVELOPED CRYSTAL DEVICE IN PARTICLE ACCELERATORS

A. Improvement of a crystal extraction/collimation of a circulating beam

Fig. 4a shows the layout of the use of a short bent crystal for beam collimation in an accelerator. Particles of a circulating beam increase the amplitude of transverse oscillations due to numerous effects, such as scattering on a residual gas, the effect of nonlinearities, and the interactions at the point of contact. As a result, the beam halo where particles fall on the leading edge of the crystal appears. Due to the channeling effect, the majority of the beam halo (fraction 1) is deeply deflected to the absorber. Only several percents of particles are deflected at an incomplete angle

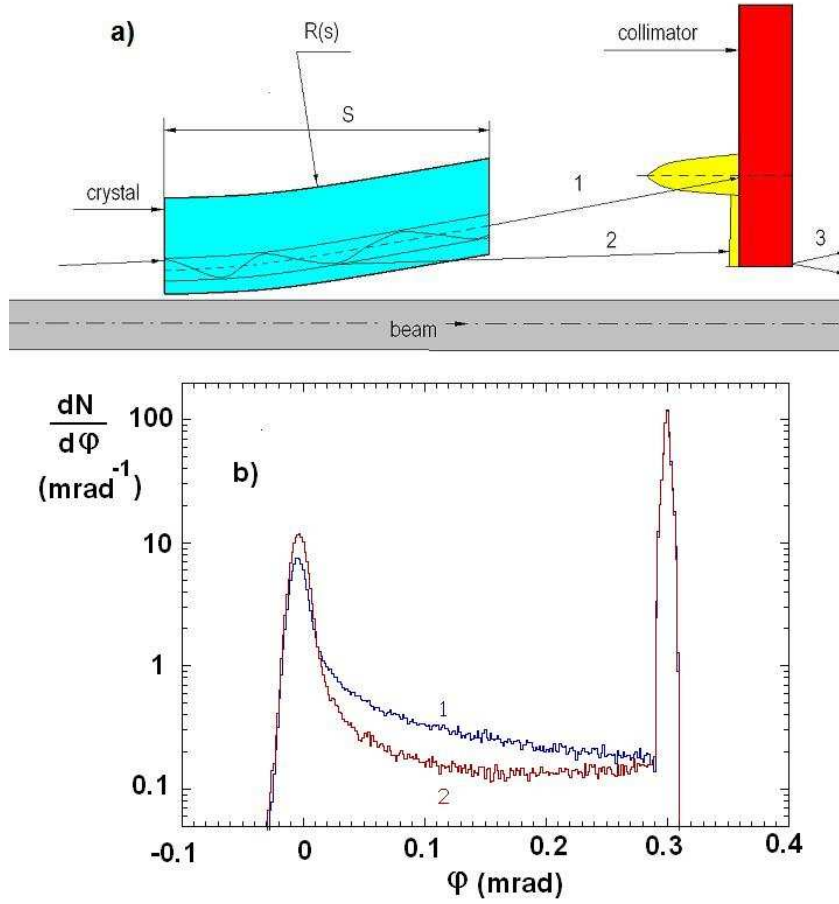


FIG. 4: A principle of beam collimation with the help of a short channeling crystal (a). Distribution of 400 GeV protons on deflection angles in 3 mm Si (111) crystal in the case of constant curvature (1) and decreasing curvature (2) (b).

due to dechanneling (fraction 2), which leads to radiation losses on the accelerator (secondary particles 3). A crystal with decreasing curvature can reduce the fraction of dechanneled particles [8]. The thing is that particles moving in the crystal are scattered from the electrons and nuclei of the lattice and, hence, some of them leave the channeling mode (dechanneling process). At the same time, due to an increase in the curvature radius along the direction of particle penetration, the available channeling region expands [8], particles move away from the maximum scattering boundary, and the fraction of dechanneled particles decreases.

That is, the fraction of the particles deflected by a crystal on an incomplete angle (it is less than bend of a crystal, see Fig.4a) decreases, that reduces losses of particles in the accelerator and improves efficiency of collimation or extraction. In Fig. 4b calculations of 400 GeV protons deflection by crystals are resulted lead by a method of Monte Carlo with use of the program described in [8, 16]. From the plots it is visible, that the fraction of "harmful" particles (between peaks of falling and fully-deflected particles) rejected in our crystal in few times in comparison with a case of uniform bent crystal. Calculations show also, that efficiency of beam extraction from U-70 can be increased from 85 % up to 95 % with new crystal device application. By corresponding optimization of parameters of the crystal, the similar device can be applied to improvement of beam collimation in the LHC.

B. Focusing of high energy particle beams

Presented in Refs. [14, 15] the consideration of focusing problem is based on a geometrical description of the process and valid only for single crystals with a constant curvature and with a special shape of skew cut of the exit face. In this paper we investigate the problem of focusing of high energy beams for a common case, in particular, for bent single crystals with a variable curvature.

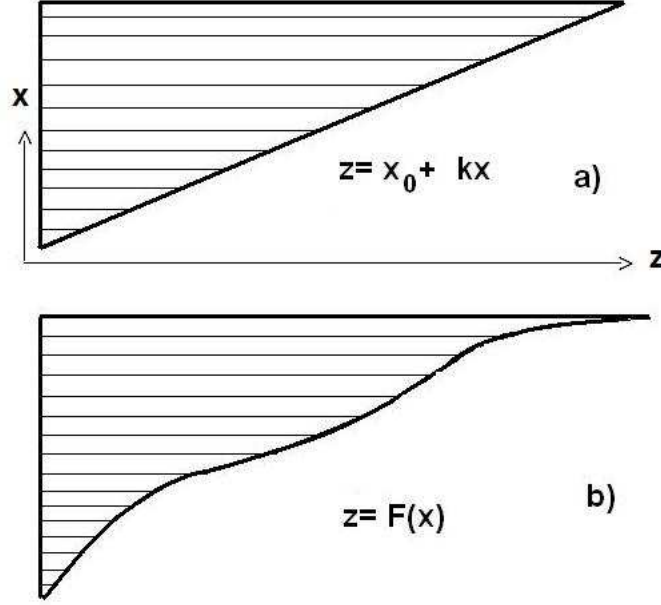


FIG. 5: Crystals with the linear (a) and arbitrary (b) forms of cut. Before usage they should be bent. With the help of different bending methods the bent crystals may be fabricated with a constant or variable curvature. One can expect that the connection $z = F(x)$ is conserved in bent crystals.

The method for calculation of beam focusing is visual and simple. Let us consider the beam channeling in a bent single crystal of constant curvature with skew cutting on the end (see, Fig. 5). On the exit from crystal particles are deflected on the angle $\varphi = L/R + \theta$, where R is the bending radius, L is the path of particle in the body of a single crystal, and θ is the angle due to oscillation motion of particles in a channeling regime. It is clear that an absolute value of θ -angle is less than the critical angle of channeling θ_c . On the first stage we put $\theta = 0$. Then in the case of single crystal without cut all the particles pass the length equal to L and, hence, they are deflected on the same angle. At the condition $\theta = 0$ the beam after end of a crystal is parallel. It is clear that for a single crystal with the cut particles with different transversal coordinates x have different paths $L(x)$ and, hence, these particles have various deflection angles. It turns out that at a special form of the cut, particles focusing on some distance from the crystal edge. We can write for the arbitrary form $F(x)$ of cut the following equation:

$$\sigma_x(l) = \int_0^d \rho(x)(X - \bar{X})^2 dx, \quad (2)$$

where $\sigma_x(l)$ and $X = x + \varphi l$ are the mean square size of beam and the particle coordinate on the distance l from a single crystal, $\rho(x)$ is the normalized (on unit) distribution function over x -coordinate (at $l = 0$) and $\bar{X} = \int_0^d \rho(x)(x + \varphi l) dx$. From here, we get

$$\sigma_x(l) = \langle x^2 \rangle - \bar{x}^2 + \langle \varphi^2 \rangle l^2 - \bar{\varphi}^2 l^2 + 2\langle x\varphi \rangle l - 2\bar{x}\bar{\varphi}l. \quad (3)$$

where $\langle x^2 \rangle$ and $\langle \varphi^2 \rangle$ are the mean square size of beam and the mean square deflection angle at $l = 0$, \bar{x} and $\bar{\varphi}$ are the mean size of beam and the mean deflection angle (at $l = 0$) and $\langle x\varphi \rangle = \int_0^d x\varphi(x)\rho(x)dx$ and d is the transversal size of a single crystal. The $\sigma_x(l)$ -function has a minimum when

$$l = l_f = -\frac{\langle x\varphi \rangle - \bar{x}\bar{\varphi}}{\langle \varphi^2 \rangle - \bar{\varphi}^2}. \quad (4)$$

The focusing takes place at the condition $l_f > 0$. It means that $\bar{x}\bar{\varphi} - \langle x\varphi \rangle > 0$. In the case when a crystal has a variable radius the deflection angle is defined by the relation:

$$\varphi(x) = \int_0^{F(x)} \frac{dz}{R(z)}. \quad (5)$$

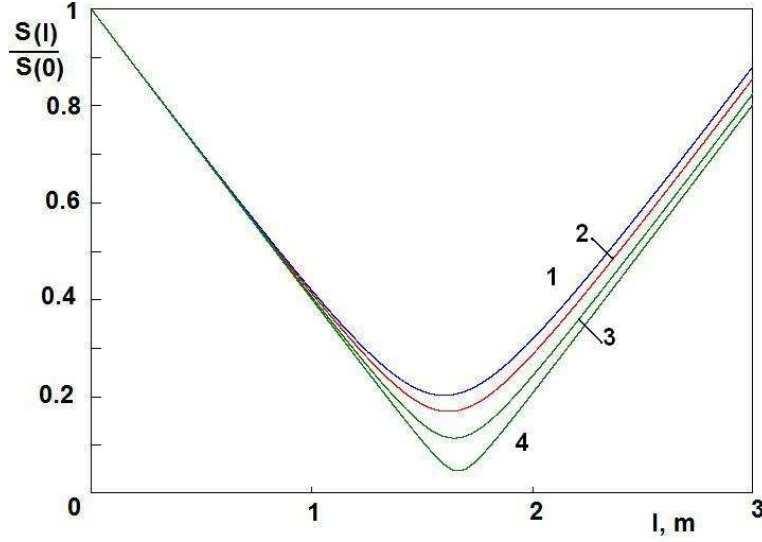


FIG. 6: The relative variation of square mean size of beam ($S(l)/S(0) = \sqrt{\sigma_x(l)/\sigma_x(0)}$) as a function of distance l . For additional information, see the text.

Here $z = F(x)$ is connection between x and z coordinates, which is determined by the form of cut. Now we can take into account the natural divergence of a beam due to oscillator motion of particles in the channeling regime. As result, we get for the total mean square size $\sigma_T(l) = \sigma_x(l) + \langle \theta^2 \rangle l^2$. We ignored the distribution over x coordinate due to a small value of atomic interplanar distance. This consideration is similar to consideration of paper [17], where the sum distribution of particles may be represented as a convolution of two independent distributions.

Let us apply our equations to a single crystal with increasing bending radius which presented in Fig. 1b. The deflection angle for a single crystal with the variation of radius defined by Eq. (1) is

$$\varphi(z) = \frac{z}{R_0} - \frac{Cz^2}{2L_0R_0}. \quad (6)$$

In accordance with Fig.1b we can take $z = F(x) = x_0 + kx$. Note that we can put $x_0 = 0$ by a corresponding choice of the coordinate system. In our further calculations we use the following quantities: $C = 0.8$, $L_0 = 0.3$ cm, $d = 0.05$ cm, $k = -6$, $R_0 = 6$ m. Note that $\varphi(x) = ax + bx^2$, where $a = k/R_0$, $b = Ck^2/(2L_0R_0)$. Taking into account a small transversal size of a crystal we take the flat distribution function over x -coordinate $\rho(x) = 1/d$. Then we get $\bar{x} = d/2$, $\langle x^2 \rangle = d^2/3$, $\bar{\varphi} = ad/2 + bd^2/3$, $\langle \varphi^2 \rangle = a^2d^2/3 + abd^3/2 + b^2d^4/5$, $\langle x\varphi \rangle = ad^2/3 + bd^3/4$. Substitution of these quantities in Eq. (3) gives the envelope of a beam. In this way we get for l_f :

$$l_f = -\frac{M_2}{M_1}, \quad (7)$$

and for the mean square beam size at $l = l_f$:

$$s_{min} = \frac{d^2}{12} - \frac{M_2^2}{M_1}, \quad (8)$$

where

$$M_1 = [a^2d^2/12 + abd^3/6 + 4b^2d^4/45 + \langle \theta^2 \rangle], \quad (9)$$

$$M_2 = ad^2/12 + bd^3/12. \quad (10)$$

It is interesting that for the linear dependence $\varphi(x) = ax$,

$$l_f = -\frac{1}{a + 12\langle \theta^2 \rangle/(ad^2)}, \quad (11)$$

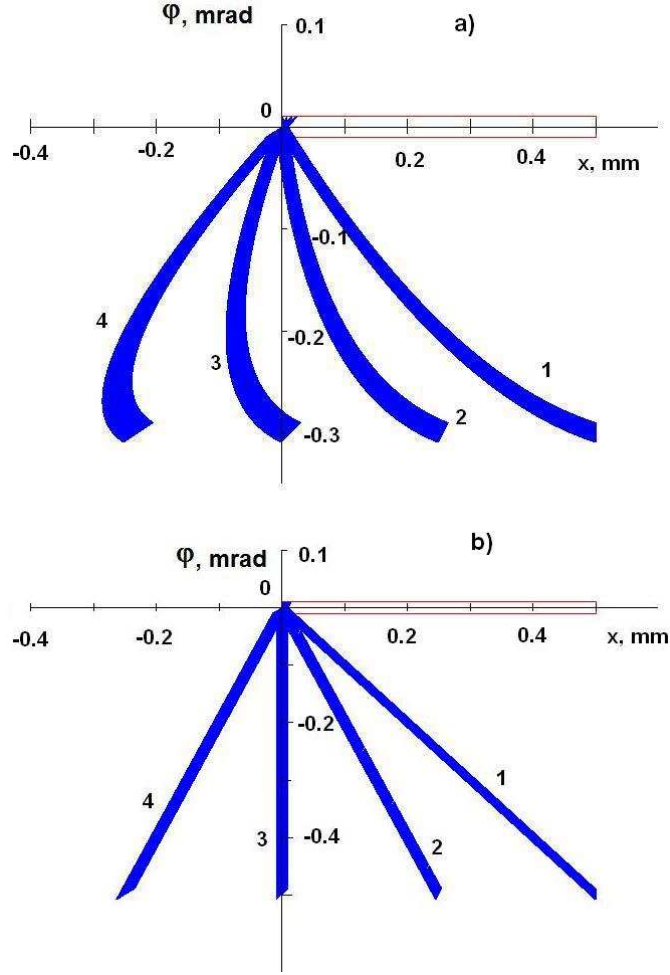


FIG. 7: The phase volume of a beam at different $l = 0$ (the curve 1), $l_f/2$ (2), l_f (3), $1.5l_f$ (4). The crystal parameters corresponds to Fig. 1b. The crystal with the linear coordinate-angle couple (b). The rectangle is the accepted phase volume. $\theta_c = 10\mu\text{rad}$.

$$s_{min} = \frac{\langle \theta^2 \rangle / a^2}{1 + 12 \langle \theta^2 \rangle / (a^2 d^2)}. \quad (12)$$

One can assume for estimations that the distribution over θ -angle is flat. It means that $\langle \theta^2 \rangle \approx \theta_c^2/6$.

Fig. 6 illustrates the behavior of square mean size of beam as a function of l for above mention crystal parameters. The curve 1 is calculated for $\theta_c = 0$ and the curve 2 is calculated for $\theta_c^2/6 = 10^{-10}$. The last value corresponds approximately to a beam energy ≈ 70 GeV. For energies ~ 400 GeV and more the both curves are very close in between. We see that $l_f \approx 1.62$ m and the minimal size of beam is in ≈ 5 times less than initial one. Note that l_f -value is proportional to the bending radius.

Fig. 7a illustrates the transformation of the two dimensional beam phase volume at different l for mention above crystal parameters. Fig. 7b illustrates the same volume but for the case when $\varphi(x) = ax$ is a linear function, From comparison of these figures it is easy to understand that for a linear angle-coordinate dependence the size of beam is less significantly, or, in other words, in this case the focusing is optimal. The same fact also follows from Eqs.(7)-(12). However, the advantage of crystal under consideration (with increasing radius) in the comparison with the linear case is more small losses of particles due to dechanneling [8].

In the papers [14, 15] a geometrical description of focusing (for crystals with a constant curvature) was considered. Here for focusing the special shape of cut was created. This shape represents the crossing of two cylindrical surfaces with different radii (R and r) One radius (R) is equal to the bending radius of crystal. However, it is easy to get that for large enough radii and thicknesses of crystal about 1 mm the couple between x and z coordinates is a linear with

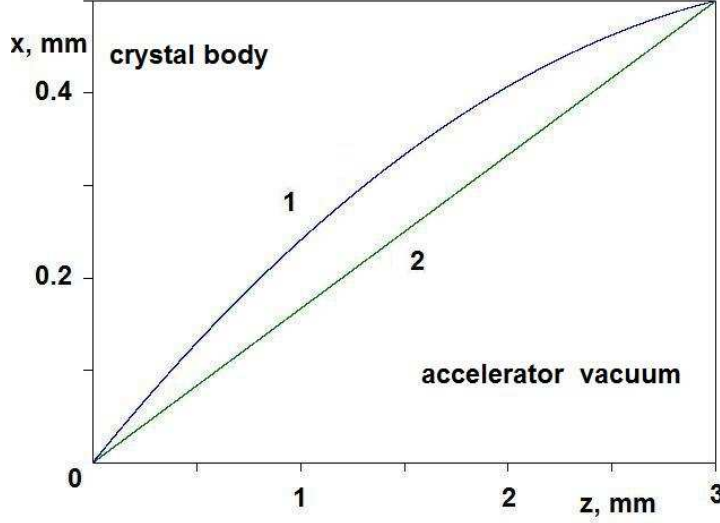


FIG. 8: Comparison of the improved form of cut (the curve 1) with linear one (the curve 2).

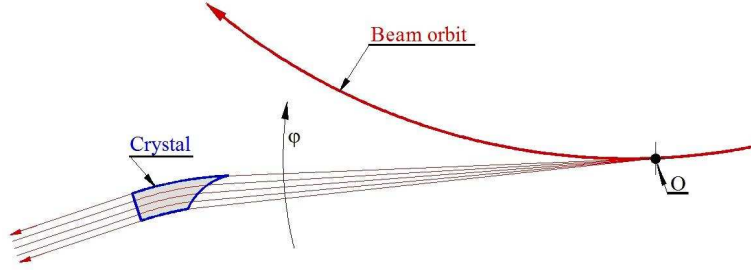


FIG. 9: The example of application of a focusing crystal for research of low-angular processes.

high accuracy. Really, we can write

$$\tan \varphi = \frac{R}{2r\sqrt{1 - R^2/(4r^2)}}, \quad (13)$$

For small thicknesses of a crystal and large enough radii we can represent $R = R_m + x$, where R_m is the minimal radius of crystal bending. Then we get the angle-coordinate couple:

$$\varphi(x) \approx \frac{R_m + x}{2r\sqrt{1 - R^2/(4r^2)}}. \quad (14)$$

From our consideration follows that for a single crystal with varied curvature and linear function of the cut $F(x)$ the focusing is not optimal. This situation may be corrected by creation of a special form of cut. Fig. 3 illustrates a good enough description of varied curvature with the help of Eq. (5) which is consistent with measurements. It is easy with the help of Eqs. (4) and (5) to find the corrected form

$$z = F(x) = \frac{L_0}{C} \left(1 - \sqrt{1 - \frac{2CR_0\kappa x}{L_0}} \right), \quad (15)$$

where $\kappa = (1 - C/2)L_0/(R_0d)$ is the coefficient in $\varphi = \kappa x$. Obviously, the function defined by Eq. (15) is independent of the bending radius. Fig. 8 illustrates this function for parameters of a crystal under consideration in the paper. The curves 3 and 4 (see Fig. 6) show the envelope of beam for corrected shape for 70 and 400 GeV protons, correspondingly.

The focusing property of the developed device can be applied on the LHC or the other accelerator of high energy to research of low-angular processes. The crystal can be align on a fix target by focusing end face, as shown in Fig. 9.

Rotating the crystal around of an axis O, one can deflect the particles from the target aside from adverse background area near the circulating beam. I.e. the role of a crystal consists in creation of clean conditions for registration of the necessary particles. Other motive of such scheme is production of the secondary particle beams in accelerators by rather simple way.

V. CONCLUSION

Developed and checked up optically the crystal device is offered to be used in experiments with crystals on accelerators of IHEP and CERN [11,12]. Two positive features characterize this device:

- the property of suppression of dechanneling for beam collimation improvement,
- the opportunity to focus the particle trajectories on small distances, which is useful tool for high energy physics.

VI. ACKNOWLEDGMENTS

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